STAT 1201/2200 Homework #4 Solution

4.2

<table>
<thead>
<tr>
<th>X_i: At Term</th>
<th>Rank(X_i)</th>
<th>Y_i: 12-26 Weeks Gestational Age</th>
<th>Rank(Y_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>3</td>
<td>1.15</td>
<td>8</td>
</tr>
<tr>
<td>0.83</td>
<td>4</td>
<td>0.88</td>
<td>5</td>
</tr>
<tr>
<td>1.89</td>
<td>14</td>
<td>0.90</td>
<td>6</td>
</tr>
<tr>
<td>1.04</td>
<td>7</td>
<td>0.74</td>
<td>2</td>
</tr>
<tr>
<td>1.45</td>
<td>11</td>
<td>1.21</td>
<td>9</td>
</tr>
<tr>
<td>1.38</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.91</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.64</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.73</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.46</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

W = 8 + 5 + 6 + 2 + 9 = 30
W' = 3 + 4 + 14 + 7 + 11 + 10 + 15 + 13 + 1 + 12 = 90
=> W + W' = 30 + 90 = 120
m : # of X observations = 10
n : # of Y observations = 5

\[
\frac{(m+n)(m+n+1)}{2} = \frac{(10+5)(10+5+1)}{2} = 120
\]

Therefore, \( W + W' = \frac{(m+n)(m+n+1)}{2} = 120 \)

4.5

<table>
<thead>
<tr>
<th>X_i: Olympics Watchers</th>
<th>Rank(X_i)</th>
<th>Y_i: Karate Kid Watchers</th>
<th>Rank(Y_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>(8)</td>
<td>37</td>
<td>(27)</td>
</tr>
<tr>
<td>44</td>
<td>(29)</td>
<td>39</td>
<td>(28)</td>
</tr>
<tr>
<td>34</td>
<td>(25)</td>
<td>30</td>
<td>(23)</td>
</tr>
<tr>
<td>14</td>
<td>(12.5)</td>
<td>7</td>
<td>(4)</td>
</tr>
<tr>
<td>9</td>
<td>(6)</td>
<td>13</td>
<td>(10)</td>
</tr>
<tr>
<td>19</td>
<td>(17)</td>
<td>139</td>
<td>(35)</td>
</tr>
<tr>
<td>156</td>
<td>(40)</td>
<td>45</td>
<td>(30)</td>
</tr>
<tr>
<td>23</td>
<td>(19.5)</td>
<td>25</td>
<td>(21)</td>
</tr>
<tr>
<td>13</td>
<td>(10)</td>
<td>16</td>
<td>(15.5)</td>
</tr>
<tr>
<td>11</td>
<td>(7)</td>
<td>146</td>
<td>(37.5)</td>
</tr>
<tr>
<td>47</td>
<td>(31)</td>
<td>94</td>
<td>(34)</td>
</tr>
<tr>
<td>26</td>
<td>(22)</td>
<td>16</td>
<td>(15.5)</td>
</tr>
<tr>
<td>14</td>
<td>(12.5)</td>
<td>23</td>
<td>(19.5)</td>
</tr>
<tr>
<td>33</td>
<td>(24)</td>
<td>1</td>
<td>(2)</td>
</tr>
<tr>
<td>15</td>
<td>(14)</td>
<td>290</td>
<td>(42)</td>
</tr>
<tr>
<td>62</td>
<td>(32.5)</td>
<td>169</td>
<td>(41)</td>
</tr>
<tr>
<td>5</td>
<td>(3)</td>
<td>62</td>
<td>(32.5)</td>
</tr>
<tr>
<td>8</td>
<td>(5)</td>
<td>145</td>
<td>(36)</td>
</tr>
<tr>
<td>0</td>
<td>(1)</td>
<td>36</td>
<td>(26)</td>
</tr>
<tr>
<td>154</td>
<td>(39)</td>
<td>20</td>
<td>(18)</td>
</tr>
<tr>
<td>146</td>
<td>(37.5)</td>
<td>13</td>
<td>(10)</td>
</tr>
</tbody>
</table>
\[ m = 21, n = 21 \]

To test \( H_0 : \Delta = 0 \) vs. \( H_1 : \Delta > 0 \)

\[
W = 27 + 28 + 23 + 4 + 10 + 35 + 30 + 21 + 15.5 + 37.5 + 34 + 15.5 + 19.5 + 2 + 42 + 41 + 32.5 + 36 + 26 + 18 + 10 = 507.5
\]

Since \( m \) and \( n \) is large, so use the large sample approximation for tied data.

\[
E_0(W) = \frac{n(m + n + 1)}{2} = \frac{21 \times (21 + 21 + 1)}{2} = 451.5
\]

\[
Var_0(W) = \frac{mn}{12} \left[ m + n + 1 - \frac{\sum_{j=1}^{q} (t_j - 1) t_j (t_j + 1)}{(m + n)(m + n - 1)} \right]
\]

\[
= \frac{21 \times 21}{12} \left[ 21 + 21 + 1 - \frac{1}{42 \times 41} (5 \times 1 \times 2 \times 3 + 2 \times 3 \times 4) \right] = 1579.098
\]

\[
W^* = \frac{W - E_0(W)}{\sqrt{Var_0(W)}} = \frac{507.5 - 451.5}{\sqrt{1579.098}} = \frac{56}{39.74} = 1.409
\]

\[ \text{P-value} = 0.0793. \]

Thus, at \( \alpha = 0.05 \), we can't reject \( H_0 \).

Therefore, there is no evidence that the children who viewed the violent TV tend to take longer to seek help than the children who viewed the nonviolent sports action TV.

### 4.6

From the equation (4.23),

\[
A_{\text{normal}} = \left( \sqrt{\frac{3mn}{(N + 1)\pi}} \cdot \frac{\Delta}{\sigma} - Z_\alpha \right)
\]

\[ m = 8, \ n = 8, \ N = 16, \ \Delta = 2, \ \sigma = \sqrt{13}, \ Z_{0.065} = 1.515 \]

\[
=> A_{\text{normal}} = \left( \sqrt{\frac{3 \times 8 \times 8}{17 \times \pi}} \cdot \frac{2}{\sqrt{13}} - 1.515 \right) = 1.0517 - 1.515 = -0.463
\]

\[ => \text{Power} = \Phi(-0.463) = 0.3228 \]

### 4.7

From the equation (4.25),

\[
N \geq \frac{(Z_\alpha + Z_\beta)^2}{12 \cdot c(1-c)} \left( \delta - \frac{1}{2} \right)^2
\]

\[ \alpha = 0.1, \ \text{power} = 1 - \beta = 0.88 => \beta = 0.12, \ \delta = P(X < Y) = 0.8 \ (\text{Recall that under } \text{H}_0, \delta = 0.5) \]

since \( m = n \) \( => \ c = 0.5, \ Z_{0.1} = 1.28, \ Z_{0.12} = 1.175 \)

Therefore, \[ N \geq \frac{(1.28 + 1.175)^2}{12(0.5)(0.5)(0.8 - 0.5)^2} = 22.32 \]

\[ m = n = \frac{N}{2} = 11.16 => \text{Therefore, to be conservative take } m = n = 12 \text{ rather than 11.} \]
For the particular observed value \( W = 13 \),

we see \( P_0(W \leq 13) = 1 - P_0(W \geq 13.5) = \frac{24}{35} \)
4.10

<table>
<thead>
<tr>
<th>$X_i$: Healthy Geese</th>
<th>Rank($X_i$)</th>
<th>$Y_i$: Lead-Poisoned Geese</th>
<th>Rank($Y_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>297</td>
<td>(8)</td>
<td>293</td>
<td>(7)</td>
</tr>
<tr>
<td>340</td>
<td>(12)</td>
<td>291</td>
<td>(6)</td>
</tr>
<tr>
<td>325</td>
<td>(10)</td>
<td>289</td>
<td>(4)</td>
</tr>
<tr>
<td>227</td>
<td>(1)</td>
<td>430</td>
<td>(14)</td>
</tr>
<tr>
<td>277</td>
<td>(3)</td>
<td>510</td>
<td>(15)</td>
</tr>
<tr>
<td>337</td>
<td>(11)</td>
<td>353</td>
<td>(13)</td>
</tr>
<tr>
<td>250</td>
<td>(2)</td>
<td>318</td>
<td>(9)</td>
</tr>
<tr>
<td>290</td>
<td>(5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$W = 7 + 6 + 14 + 15 + 13 + 9 = 68$

To test $H_0 : \Delta = 0$ vs $H_1 : \Delta > 0$

at the $\alpha$ level of significance, Reject $H_0$ if $W \geq W_\alpha$

$P-value = P(W \geq 68) = 0.095$ is obtained by entering Table A.6 at $n = 7, m = 8$.

For $\alpha = 0.047$, $W_\alpha = 71$,
For $\alpha = 0.116$, $W_\alpha = 67$

Since $P-value 0.095$ is between 0.05 and 0.10, thus we would reject $H_0$ for $\alpha = 0.10$, but not for $\alpha = 0.05$.

4.12

Let $X'$'s ranks are $1, 2, \cdots, m$ and $Y'$'s ranks are $m + 1, m + 2, \cdots, m + n$.

i.e. all ranks of $Y$ are greater than $X'$'s.

Then, $W = (m + 1) + (m + 2) + \cdots + (m + n) = m \cdot n + \frac{n(n + 1)}{2} = \frac{n(2m + n + 1)}{2}$.

Let $Y'$'s ranks are $1, 2, \cdots, n$ and $X'$'s ranks are $n + 1, n + 2, \cdots, n + m$

i.e. all ranks of $Y$ are less than $X'$'s.

Then, $W = \frac{n(n + 1)}{2}$.

Therefore, minimum value of $W$ is $\frac{n(n + 1)}{2}$.