4.18

From (4.16), we have \( W = U + \frac{(n)(n+1)}{2} \) and \( U = W - \frac{(n)(n+1)}{2} = 151 - \frac{9(10)}{2} = 106 \)

Hence, we estimate \( \delta \) by \( \hat{\delta} = \frac{U}{mn} = 106 \times \frac{1}{3960} = 0.0960 \).

Sen’s asymptotic interval for \( \delta \) is \( \hat{\delta} \pm Z_{\alpha/2} \sqrt{\frac{n S_{10}^2 + m S_{01}^2}{nm}} \), where

\[
\sum_{j=1}^{9} S_j = 151, \quad \sum_{j=1}^{13} R_j = 102, \quad \bar{R} = 7.846, \quad \bar{S} = 16.778, \quad \sum_{i=1}^{13} (R_i - i)^2 = 19, \quad \sum_{j=1}^{9} (S_j - j) = 1300
\]

\[
S_{10}^2 = \frac{1}{(m-1)n^2} \left[ \sum_{i=1}^{13} (R_i - i)^2 - m \left( \bar{R} - \frac{m+1}{2} \right)^2 \right] = 0.00997
\]

\[
S_{01}^2 = \frac{1}{(n-1)m^2} \left[ \sum_{j=1}^{9} (S_j - j)^2 - n \left( \bar{S} - \frac{n+1}{2} \right)^2 \right] = 0.0381
\]

With \( \alpha = 0.1 \), we find \( \hat{\delta} \pm Z_{0.05} \sqrt{\frac{n S_{10}^2 + m S_{01}^2}{nm}} = 0.906 \pm 1.645 \times 0.707 = (0.790, 1.023) \)

4.19

With \( m = n = 21 \), there are total of \( mn = 21^2 = 441 \) differences of \( \{Y_j - X_i\} \).

\( \hat{\Delta} = \text{median}\{Y_j - X_i, 1 \leq i \leq 21, 1 \leq j \leq 21\} = U(221) = 10 \)

4.21

Using comment 17,

\( \hat{\Delta} = \text{median}\{\frac{Y_i + Y_j}{2}, 1 \leq i \leq j \leq n\} - \text{median}\{\frac{X_i + X_j}{2}, 1 \leq i \leq j \leq m\} = 47.5 - 26 = 21.5 \)

Set \( \alpha_1 = \alpha_2 = 0.035 \) (\( \alpha_1 + \alpha_2 = \alpha \)) then \( \theta_{1L} \) and \( \theta_{1U} \) are the endpoints of 100(1 - \( \alpha_1 \))% Confidence interval obtained from the method of section 3.3 by the \( mX \)’s and \( \theta_{2L} \) and \( \theta_{2U} \) are the endpoints of 100(1 - \( \alpha_2 \))% Confidence interval obtained from the method of section 3.3 by the \( nY \)’s.

Thus, \( \theta_{1L} = 14, \theta_{1U} = 78.5, \theta_{2L} = 24, \theta_{2U} = 91. \)

\[
\hat{\sigma}_{\hat{\Delta}} = \left( \frac{\theta_{2U} - \theta_{2L}}{2 \times Z_{\alpha/2}} \right)^2 + \left( \frac{\theta_{1U} - \theta_{1L}}{2 \times Z_{\alpha/2}} \right)^2 = 22.037
\]

\[
\hat{\Delta} - Z_{\alpha/2} \times \hat{\sigma}_{\hat{\Delta}} = 21.5 - 1.81 \times 22.037 = -18.38, \quad \hat{\Delta} + Z_{\alpha/2} \times \hat{\sigma}_{\hat{\Delta}} = 21.5 + 1.81 \times 22.037 = 61.38
\]

Thus, an approximate 93% confidence interval for \( \Delta \) is \((-18.38, 61.38)\)
4.28

With $\alpha=0.96$, $m=10$, and $n=5$, we have from Table A.5 that $w\left(\frac{a}{2}, m, n\right) = 57$ and obtain $C_a = \frac{1}{2}5(20 + 5 + 1) - 57 = 9$. Hence, $\Delta_L = U^{(9)} = -0.76$, and $\Delta_U = U^{(42)} = 0.15$. Therefore, by Comment(4.20), we can estimate $\Delta$ by $\hat{\Delta} = \frac{1}{2}[U^{(9)} + U^{(42)}] = -0.305$, which is same as $\hat{\Delta} = -0.305$ obtained from Example 4.3.

4.33

For $\alpha=0.1$, $m=12, n=11, Z_{0.05} = 1.645$.

$C_a \approx \frac{mn}{2} - Z_\alpha \left(\frac{mn(m + n + 1)}{12}\right)^{\frac{1}{2}} = \frac{12 \times 11}{2} - 1.645 \left(\frac{12 \times 11 \times 24}{12}\right)^{\frac{1}{2}} = 66 - 26.728 = 39.272 \approx 39$

$\Delta_L = U^{(39)} = -678$, and $\Delta_U = U^{(94)} = 217$

Thus 90% confidence interval for $\Delta$ is (-678, 217).

5.3

To test $H_0 : \gamma^2 = 1$ vs $H_1 : \gamma^2 < 1$

$C = \sum_{j=1}^{13} R_j = 102, N = m+n = 21+21=42$ is even.

$E_0(C) = \frac{n(N + 2)}{4} = \frac{21 \times (42 + 2)}{4} = 231$

$Var_0(W) = \frac{mn\left[16 \sum_{j=1}^{g} t_j \eta_j^2 - N(N + 2)\right]}{16 \times N \times (N - 1)} = 393.2$

$C^* = \frac{C - E_0(C)}{\sqrt{Var_0(W)}} = \frac{239.5 - 231}{\sqrt{393.2}} = 0.429$

$p\text{-value} = P(C^* \leq 0.429) = 1 - P(C^* \geq 0.429) = 1 - 0.3336 - 0.6664$